▸ VLADIMIR KANOVEI, *The full basis theorem does not imply analytic wellordering.*
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Let *analytically definable* mean lightface Σ^1_n for some n.

THEOREM 1 (with Vassily Lyubetsky, ArXived in [6]). *In a suitable ccc generic extension of L, it is true that every non-empty analytically definable set of reals contains an analytically definable real (the full basis theorem), but there is no analytically definable wellordering of the continuum.*

To prove the theorem, we define, in L, a system of forcing notions $P_{\xi k}$, $\xi < \omega_1$ and $k < \omega$, whose finite-support product $P = \prod_{\xi,k} P_{\xi k}$ adds an array $X = \langle x_{\xi k}\rangle_{\xi<\omega_1 \wedge k<\omega}$ of reals $x_{\xi k} \in 2^\omega$ to L, such that the following holds in $L[X]$:

(1) if $m < \omega$ then the submodel $L[X_m]$ admits a Δ^1_{m+3} wellordering of the reals of length ω_1, where $X_m = \langle x_{\xi k}\rangle_{\xi<\omega_1 \wedge k<m}$;
(2) if $m < \omega$ then $2^\omega \cap L[X_m]$ is a Σ^1_{m+3} set in $L[x]$;
(3) if $m < \omega$ then $L[X_m]$ is an elementary submodel of $L[x]$ with respect to all Σ^1_{m+2} formulas with reals in $L[X_m]$ as parameters;
(4) there is no analytically definable wellordering of 2^ω.

Each factor $P_{\xi k}$ of P is similar to the Jensen minimal Π^1_2 singleton forcing [3] to some extent, but corresponds to a definability level which depends on k (rather than just Π^1_2 for all k and ξ). See [2, 4, 5] on other results by the same method.

Infinite finite-support products of Jensen-type forcing notions were introduced and conjectured to be applicable to studies of definability problems by Ali Enayat [1], whose advise, as well as support of Department of Philosophy, Linguistics and Theory of Science at the University of Gothenburg and the Erwin Schrodinger International Institute for Mathematics and Physics (ESI) at Vienna, are thankfully acknowledged.

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[5] ———, *Counterexamples to countable-section Π^1_2 uniformization and Π^1_3 separation*, ***Annals Pure and Applied Logic***, vol. 167 (2016), 3, pp. 262–283.
[6] ———, ***The full basis theorem does not imply analytic wellordering***, ArXiv e-print: 1702.03566, 2017.