NEW FEATURES IN MATHPARTNER 2021

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К ДИСКУССИИ

MathPartner – это то, что должно прийти в школу вместе с

(1) Базой учебников,
(2) Базой самостоятельных и контрольных работ
(3) Кабинетами для Ученика, Учителя, Директора, Министра образования.
(4) ЕГЭ экзамен по математике и физике будет происходить интерактивно
out :

PLAN

New functions:

(1) arithmetic-geometric mean
(2)geometric-harmonic mean
(3) modified arithmetic-geometric mean proposed by Semjon Adlaj
(4) Sylvester matrices of the first and the second kind

Today's list of matrix functions:

inverse, adjugate, conjugate, transpose,
generalized inverse,
pseudo inverse,
determinant, kernel, echelon form,
characteristic polynomial,
Bruhat decomposition, LDU decomposition,

Numerical matrix functions:

QR decomposition,
SVD decomposition,

Cholesky decomposition
Linear programming

see at: mathpar.ukma.edu.ua and mathpar.com

$out:$

3.141592653589793238462643

SIX MEANS AND THE COMPLETE ELLIPTIC INTEGRALS

Given two non-negative numbers x and y, we can define their
(1) arithmetic: $\frac{x+y}{2}$,
(2) geometric: $\sqrt{xy}$,
(3) harmonic: $\frac{2xy}{x+y}$ means.

(4) $\mathbf{AGM(x,y)}$ denotes the arithmetic-geometric mean.

(Johann Carl Friedrich Gauss at the end of the 18th century).

(5) $\mathbf{GHM(x,y)}$ denotes the geometric-harmonic mean.
(6) $\mathbf{MAGM}(x,y)$ denotes the modified arithmetic-geometric mean.

It is defined bySemjon Adlaj.

Every mean is a symmetric homogeneous function in their two variables x and y.

$\mathbf{AGM(x,y)}$ is equal to the limit of both sequences x_n and y_n, where $x_0 = x$, $y_0 = y$, $x_{n+1} = \frac{1}{2}(x_n + y_n)$, and $y_{n+1} = \sqrt{x_n y_n}$.

$\mathbf{GHM}(x,y)$ is equal to the limit of both sequences x_n and y_n, where $x_0 = x$, $y_0 = y$, $x_{n+1} = \sqrt{x_n y_n}$, and $y_{n+1} = \frac{2x_n y_n}{x_n + y_n}$. Note that $\mathbf{AGM(x,y)GHM(x,y) = xy}$.

$\mathbf{MAGM(x,y)}$ is equal to the limit of the sequence x_n, where $x_0 = x$, $y_0 = y$, $z_0 = 0$, $x_{n+1} = \frac{x_n+y_n}{2}$, $y_{n+1} = z_n + \sqrt{(x_n - z_n)(y_n - z_n)}$, and $z_{n+1} = z_n - \sqrt{(x_n - z_n)(y_n - z_n)}$.

EXAMPLE:
- - - - -
$SPACE = R64[]; FLOATPOS = 4;$
$a = \mathbf{AGM}(1,5); g = \mathbf{GHM}(1,5); m = \mathbf{MAGM}(1,5);$
$[a, g, m]$
$out:$

$[2.604, 1.9201, 2.6105]$

Elliptic integrals

These means are applicable, in particular, to calculate the complete elliptic integrals of the first and second kind. Let us use the parameter $0 \leq k \leq 1$.

The complete elliptic integral of the first kind $K(k)$ is defined as

$$K(k) = \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}}$$

It can be computed in terms of the arithmetic-geometric mean:

$$K(k) = \frac{\pi}{2\mathbf{AGM}(1, \sqrt{1-k^2})}$$

On the other hand, for $k < 1$, it can be computed in terms of the geometric-harmonic mean:

$$K(k) = \frac{\pi}{2}\mathbf{GHM}(1, \frac{1}{\sqrt{1-k^2}})$$

The complete elliptic integral of the second kind $E(k)$ is defined as

$$E(k) = \int_0^1 \sqrt{\frac{1-k^2t^2}{1-t^2}}dt$$

It can be computed in terms of the modified arithmetic-geometric mean:

$$E(k) = K(k)\mathbf{MAGM}(1, 1-k^2)$$

The circumference of an ellipse is equal to

$$2\pi\frac{\mathbf{MAGM}(a^2, b^2)}{\mathbf{AGM}(a, b)},$$

where the semi-major and semi-minor axes are denoted a and b.
On the other hand, π can be expressed as

$$\pi = \frac{(\mathbf{AGM}(1, \sqrt{2}))^2}{\mathbf{MAGM}(1, 2) - 1}$$

So, to calculate π one can run the commands

- - -

$SPACE = R[]; FLOATPOS = 24;$
$w = \sqrt(2);$
$Pi = \ \mathbf{AGM}(1, w)^2/(\mathbf{MAGM}(1, 2) - 1);$
$pi = \ \mathbf{value}(\pi); \ \mathbf{print}(P, pi);$
$out:$

$P = 3.141592653589793238462643$
$pi = 3.141592653589793238462643$

The period of the pendulum

Let a point mass be suspended from a pivot with a massless cord. The length of the pendulum is denoted by L. It swings under gravitational acceleration $g = 9.80665 m/s^2$. The maximum angle that the pendulum swings away from the vertical, called the amplitude, is denoted by θ_0.

One can find the period T of the pendulum using the arithmetic-geometric mean

$$T = \frac{2\pi}{\mathbf{AGM}(1, \ \mathbf{cos}(\theta_0/2))}\sqrt{\frac{L}{g}}$$

If $L = 1$ m and $\theta_0 = 120°$, then $T = 2.7546$ s.
To calculate the period one can run the commands

- - - -

$SPACE = R64[]; FLOATPOS = 4; L = 1; g = 9.80665;$
$\mathbf{value}(2 \cdot \pi \cdot \sqrt{L/g}/\ \mathbf{AGM}(1, 0.5))$
$out:$

THE SYLVESTER MATRICES, THE RESULTANT, AND THE DISCRIMINANT

Let us consider two univariate polynomials

$$f(x) = f_n x^n + ... + f_1 x + f_0, \quad g(x) = g_m x^m + ... + g + 1x + g_0,$$

where $\mathbf{deg}(f) = n$, $\mathbf{deg}(g) = m$, and $m \leq n$ hold. James Joseph Sylvester introduced two matrices associated to $f(x)$ and $g(x)$. More precisely, there are two different Sylvester matrices associated with two univariate polynomials. Let us denote

The Sylvester matrix of the first kind was introduced in 1840. It is the $(n+m) \times (n+m)$ matrix.

Its determinant is called the **resultant** of f and g.

For example, if $f = x^3 + px + q$ and $g = 3x^2 + p$, then the Sylvester matrix of the first kind is equal to

$$\begin{pmatrix} 1 & 0 & p & q & 0 \\ 0 & 1 & 0 & p & q \\ 3 & 0 & p & 0 & 0 \\ 0 & 3 & 0 & p & 0 \\ 0 & 0 & 3 & 0 & p \end{pmatrix}$$

and its determinant equals $4p^3 + 27q^2$, i.e., it is the opposite of the discriminant of f.

The Sylvester matrix of the second kind was introduced in 1853 as an improvement of the Sturm theory. It is the $(2n) \times (2n)$ matrix, where $n \geq m$. The first and the second rows are

$$\begin{pmatrix} f_n & ... & f_{m+1} & f_m & ... & f_0 & 0 & ... & 0 \\ 0 & ... & 0 & g_m & ... & g_0 & 0 & ... & 0 \end{pmatrix}$$

The next pair is the first pair, shifted one column to the right; the first elements in the two rows are zero. The remaining rows are obtained the same way as above.

For example, if $f = x^3 + px + q$ and $g = 3x^2 + p$, then the Sylvester matrix of the second kind is equal to

$$\begin{pmatrix} 1 & 0 & p & q & 0 & 0 \\ 0 & 3 & 0 & p & 0 & 0 \\ 0 & 1 & 0 & p & q & 0 \\ 0 & 0 & 3 & 0 & p & 0 \\ 0 & 0 & 1 & 0 & p & q \\ 0 & 0 & 0 & 3 & 0 & p \end{pmatrix}$$

The Sylvester matrix can be calculated in MathPartner
$\backslash sylvester(f, g, kind)$, $kind$=0 or 1.

The resultant of two univariate polynomials can be calculated as
$\backslash resultant(f, g)$.
EXAMPLE:
- - - -
$SPACE = Z[a, b, c, x]; f = a \cdot x^2 + b \cdot x + c; g = 2 \cdot a \cdot x + b;$
$\mathbf{resultant}(f, g);$
$out:$

$4ca^2 - b^2 a$

The **discriminant** of a univariate polynomial

$$f(x) = f_d x^d + ... + f_0$$

is equal to

$$\textbf{discriminant}(f) = \frac{(-1)^{d(d-1)/2}}{f_d}\textbf{resultant}(f, D(f))$$

EXAMPLE
- - - -
$SPACE = Z[a, b, c, x];$
$f = a \cdot x^2 + b \cdot x + c;$
$\textbf{discriminant}(f);$
$out:$

$-4ca + b^2$