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On binary solutions to a system of linear equations over a computable field

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Let us consider the recognition problem whether there is a $(0,1)$-solution to a system of m (inhomogeneous) linear equations

$$\begin{cases} x_j &= \ell_j(x_1,\dots,x_s), j>s,\\ x_i &\in \{0,1\}, i>0, \end{cases}$$

where ℓ_j denote linear functions over a field K of characteristic $\mathrm{char}(K)\neq 2$. Let the field K be infinite and computable in polynomial time. The worst-case complexity of such problems may be much greater than the generic-case complexity.

From a geometric point of view, we consider the recognition problem whether a given s-dimensional affine subspace passes through a vertex of the multidimensional unit cube. Our heuristic algorithm tries to find a hypersurface of degree d that passes through every vertex of the cube but does not intersect the given affine subspace. If the hypersurface is found, then the algorithm rejects the input. Else it halts in the vague halting state. In any case, the algorithm does not make any error. It does not accept any input. But the algorithm is useful because, under the described conditions, uncertainty occurs on a small fraction of inputs among all inputs of a given size.

Let us denote by $u(d,s)$ the sum of the binomail coefficients:

$$u(d,s)=1+s+\binom{s}{2}+\dots+\binom{s}{d}.$$

For a given value of the parameter $d\geq 2$, our algorithm is based on checking the solvability of an auxiliary system of $u(d,s)$ linear equations in $u(d-2,s)m$ variables.

Theorem. *Given positive integers s, m_1, m_2, and $d\geq 2$. Let both inequalities $m_1(d-1)\geq s-d+2$ and $m_2d(d-1)\geq(s-d+2)(s-d+1)$ hold. For any $\varepsilon>0$, let us consider a list of $m=1+m_1+m_2$ linear functions $\ell_j(x_1,\dots,x_s)$, where coefficients are independently and uniformly distributed on a set of $\lceil 2u(d,s)/\varepsilon\rceil$ elements of the field K. The proposed algorithm rejects the input with a probability at least $1-\varepsilon$. If a $(0,1)$-solution exists, then the algorithm does not reject the input.*

So, we improve sufficient conditions under which the generic-case complexity of the problem is subexponential. The case $d=2$ has been considered earlier [1].

References

[1] Seliverstov A.V., Binary solutions to large systems of linear equations. Prikladnaya Diskretnaya Matematika, no. 52, 5–15 (2021). (in Russian) https://doi.org/10.17223/20710410/52/1

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