Mathematicae, vol. 192 (2006), pp. 37–65.

[3] Randall Holmes, *Strong Axioms of Infinity in NFU*, ***Journal of Symbolic Logic***, vol. 66 (2001), no. 1, pp. 87–116.

[4] Ronald B. Jensen, *On the Consistency of a Slight (?) Modifcation of Quine's New Foundations*, ***Synthese***, vol. 19 (1969), no. 1, pp. 250–263.

▸ JUAN CARLOS MARTÍNEZ, *On thin-tall and thin-thick Boolean spaces.*

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Recall that a topological space X is *scattered.* if every non-empty subspace of X has an isolated point. If X is a scattered Boolean space and α is an ordinal, we denote by $I_\alpha(X)$ the α^{th}-Cantor-Bendixson level of X, i.e. $I_\alpha(X) =$ set of isolated points of $X \setminus \bigcup\{I_\beta(X) : \beta < \alpha\}$. The *height of* X is defined by $\mathrm{ht}(X) =$ the least ordinal α such that $I_\alpha(X)$ is finite. And *the cardinal sequence of* X is defined by $\mathrm{CS}(X) = \langle |I_\alpha(X)| : \alpha < \ \mathrm{ht}(X)\rangle$.

If α is an ordinal, we put $\mathcal{C}(\alpha) = \{\mathrm{CS}(X) : X$ is a scattered Boolean space of height $\alpha\}$. If κ is an infinite cardinal and α is an ordinal, we denote by $\langle\kappa\rangle_\alpha$ the constant κ sequence of length α. And if f and g are sequences of infinite cardinals, we denote by $f \frown g$ the concatenation of f with g. If X is a scattered Boolean space and κ is an infinite cardinal, we say that X is κ*-thin-tall*, if $\mathrm{CS}(X) = \langle\kappa\rangle_\alpha$ for some ordinal $\alpha \geq \kappa^+$. And we say that X is κ*-thin-thick*, if $\mathrm{CS}(X) = \langle\kappa\rangle_\kappa \frown \langle\lambda\rangle$ for some cardinal $\lambda > \kappa$.

It is well-known that $\langle\omega\rangle_\alpha \in \mathcal{C}(\alpha)$ for every ordinal $\alpha < \omega_2$ and that it is relatively consistent with ZFC that $\langle\omega\rangle_\alpha \in \mathcal{C}(\alpha)$ for every ordinal $\alpha < \omega_3$. Also, it was shown by Baumgartner that $\langle\omega_1\rangle_{\omega_1} \frown \langle\omega_2\rangle \notin \mathcal{C}(\omega_1 + 1)$ in the Mitchell Model. And it was shown by Koepke and Martínez that if $V = L$ holds then for every regular cardinal κ, $\langle\kappa\rangle_{\kappa^+} \in \mathcal{C}(\kappa^+)$ and $\langle\kappa\rangle_\kappa \frown \langle\kappa^+\rangle \in \mathcal{C}(\kappa+1)$. However, no result is known on the existence of κ-thin-tall or κ-thin-thick spaces where κ is a singular cardinal.

Then, we shall present here a general construction of scattered Boolean spaces with a large top. As consequences of this construction, we obtain the following results:

1. If κ is a singular cardinal of cofinality ω, then $\langle\kappa\rangle_\kappa \frown \langle\kappa^\omega\rangle \in \mathcal{C}(\kappa + 1)$.
2. If κ is an inaccessible cardinal, then $\langle\kappa\rangle_\kappa \frown \langle\kappa^\kappa\rangle \in \mathcal{C}(\kappa + 1)$.
3. If GCH holds, then for every infinite cardinal κ we have $\langle\kappa\rangle_\kappa \frown \langle\kappa^{\mathrm{cf}(\kappa)}\rangle \in \mathcal{C}(\kappa+1)$.

Also, we shall present some results and open problems on the existence of thin-tall spaces in relation to large cardinals.

▸ VLADIMIR KANOVEI, *Some applications of finite-support products of Jensen's minimal Δ^1_3 forcing.*

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Jensen [5] introduced a forcing notion $P \in \mathrm{L}$ such that any P-generic real a over L has minimal L-degree, is Δ^1_3 in $\mathrm{L}[a]$, and is the only P-generic real in $\mathrm{L}[a]$. Further applications of this forcing include iterations [1], finite products and finite-support infinite products for symmetric choiceless models [2, 4], et cetera. We present some new applications of finite-support infinite products of Jensen's forcing and its variations.

Theorem 1 (with V. Lyubetsky). *There is a generic extension* $\mathrm{L}[a]$ *of* L *by a real in which* $[a]_{\mathsf{E}_0}$ *is a countable lightface* Π^1_2 *set not containing any ordinal-definable reals.*

Recall that E_0 is an equivalence relation on ω^ω such that $x \mathrel{\mathsf{E}_0} y$ iff $x(k) = y(k)$ for all but finite k, and $[a]_{\mathsf{E}_0} = \{b \in \omega^\omega : a \mathrel{\mathsf{E}_0} b\}$ is the (countable) E_0-class of a real $a \in \omega^\omega$.

Let a *Groszek – Laver pair* be any OD (ordinal-definable) pair of sets $X, Y \subseteq \omega^\omega$ such that neither of X, Y is separately OD. As demonstrated in [3], if $\langle x, y\rangle$ is a Sacks×Sacks generic pair of reals over L then their L-degrees $X = [x]_{\mathrm{L}} \cap \omega^\omega$ and $Y = [y]_{\mathrm{L}} \cap \omega^\omega$ form such a pair in $\mathrm{L}[x, y]$; the sets X, Y is this example are obviously uncountable.

Theorem 2 (with M. Golshani and V. Lyubetsky). *There is a generic extension* $\mathrm{L}[a, b]$ *of* L *by reals* a, b *in which it is true that the countable sets* $[a]_{\mathsf{E}_0}$ *and* $[b]_{\mathsf{E}_0}$ *form a Groszek – Laver pair, and moreover the union* $[a]_{\mathsf{E}_0} \cup [b]_{\mathsf{E}_0}$ *is a lightface* Π^1_2 *set.*

Theorem 3 (with V. Lyubetsky). *It is consistent with* **ZFC** *that there exists a lightface* Π^1_2 *set* $\emptyset \neq Q \subseteq \omega^\omega \times \omega^\omega$ *with countable cross-sections* $Q_x = \{y : \langle x, y\rangle \in Q\}$, $x \in \omega^\omega$, *non-uniformizable by any ROD set. In fact each cross-section* Q_x *in the example is a* E_0 *class.*

ROD = real-ordinal-definable. Typical examples of non-ROD-uniformizable sets, like $\{\langle x, y\rangle : y \notin \mathrm{L}[x]\}$ in the Solovay model, definitely have **un**countable cross-sections.

[1] Uri Abraham, *A minimal model for* $\neg CH$ *: iteration of Jensen's reals*, ***Transactions of the American Mathematical Society***, vol. 281 (1984), pp. 657–674.

[2] Ali Enayat, *On the Leibniz-Mycielski axiom in set theory*, ***Fundamenta Mathematicae***, vol. 181 (2004), no. 3, pp. 215–231.

[3] M. Groszek and R. Laver, *Finite groups of OD-conjugates*, ***Periodica Mathematica Hungarica***, vol. 18 (1987), pp. 87–97.

[4] Vladimir Kanovei, *On the nonemptiness of classes in axiomatic set theory*, ***Mathematics of the USSR-Izvestiya***, vol. 12 (1978), no. 3, pp. 507–535.

[5] Ronald Jensen, *Definable sets of minimal degree*, ***Mathematical Logic and Foundations of Set Theory, Proceedings of an International Colloquium*** (Jerusalem 1968), (Yehoshua Bar-Hillel, editor), North-Holland, 1970, pp. 122–128.

CONTRIBUTED TALKS 4

Tuesday, 4 August • 4.00PM–6.00PM
Venue - PIV

▸ AHMAD KARIMI, *Yablo's paradox(es) as theorem(s) in temporal Logic.*

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This is a joint work with Saeed Salehi.

Paradoxes are interesting puzzles in philosophy and mathematics. They can be more interesting when they turn into genuine theorems. For example, Russell's paradox which collapsed Frege's foundations of mathematics, is now a classical theorem in set theory, implying that no set of all sets can exist. Or, as another example, the Liar paradox has turned into Tarski's theorem on the undefinability of truth in sufficiently rich languages. This paradox also appears implicitly in the proof of Gödel's first incompleteness theorem. For this particular theorem, some other paradoxes such as Berry's ([1, 2]) or Yablo's ([7, 8]) have been used to give alternative proofs ([4, 6]). A more recent example is the surprise examination paradox [3] that has turned into a beautiful proof for Gödel's second incompleteness theorem ([5]). In this talk, we transform Yablo's paradox into a theorem in the Linear Temporal Logic (LTL). This paradox, which is the first one of its kind that supposedly avoids self–reference and circularity has been used for proving an old theorem ([4, 6]) but not a new theorem had been made out of it. Here, for the very first time, we use this paradox (actually its argument) for proving some genuine mathematical theorems in LTL. The thought is that we can make progress by thinking of the sentences in the statement of Yablo's paradox not as an infinite family of atomic propositions but as a single proposition evaluated in lots of worlds in a Kripke model. Thus the derivability of Yablo's paradox should be the same fact as the theoremhood of a particular formula in the linear temporal logic. This temporal treatment also unifies other versions of Yablo's paradox.