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AN ANSWER TO LUZIN'S QUESTION ABOUT THE SEPARABILITY OF CA-CURVES

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We obtain an affirmative answer to the following question, posed by Luzin [1]: Do there exist two CA-curves, one of which lies under the other, that are nonseparable B by means of a set?

We give some definitions [1]. An everywhere-defined function y = f(x) on OX whose graph is a CA-set (as a subset of the plane OXY) is called a CA-curve in the plane OXY. See, e.g., [2, p. 586] for the definition of the classes B (of the Borel sets), A (of the analytic sets), and CA (of the analytic complements).

A curve y = f(x) lies under a curve y = g(x) if f(x) < g(x) for all points x of the axis OX.

Let $E \subseteq OXY$. Each point x of the axis OX determines the section $E_x = \{y: \langle x, y\rangle \in E\}$ of the set E by the ordinate with abscissa x. The projection of E on the axis OX is the set Pr E = {x: E_x is nonempty}.

Luzin [1] called two CA-curves y = f(x) and y = g(x), the first of which lies under the second, separable B by means of a set if there exists a plane Borel set E such that Pr E = OX and f(x) < y < g(x) whenever $x \in OX$ and $\langle x, y\rangle \in E$.

THEOREM 1. There exist two CA-curves y = f(x) and y = g(x), the first of which lies under the second, that are not separable B by means of a set.

We start the proof of this theorem by fixing a CA-curve y = f(x) that is not a B-curve (this means that the graph $\{\langle x, f(x)\rangle: x \in OX\}$ is not a Borel set). The existence of such curves has been proved in [1].

For all $n \in \omega$ and $x \in OX$ we set $g_n(x) = f(x) + 2^{-n}$. We have the family of the CA-curves y = $g_n(x)$ (each of them is, in fact, a CA-curve, since it is obtained from the CA-curve y = f(x) by vertical parallel translation). In addition, the curve y = f(x) lies under each of the curves y = $g_n(x)$.

Now to prove Theorem 1 it is sufficient to prove the following lemma.

LEMMA 1. There exists an n such that the curves y = f(x) and y = $g_n(x)$ are not separable B by means of a set.

Proof. Let us assume the contrary. Then for each n there exists a plane Borel set E_n with Pr E_n = OX such that $f(x) < y < f(x) + 2^{-n}$ whenever $\langle x, y\rangle \in E_n$.

In order to obtain a contradiction, let us consider the sets $W_n = \{\langle x, y\rangle$: there exists a $y' \in E_{nx}$, such that $|y - y'| < 2^{-n}\}$.

LEMMA 2. Each W_n is an A-set.

Proof. The proof is simple. W_n is the projection on the plane OXY of the space Borel set

$$\{\langle x, y, z\rangle: \langle x, z\rangle \in E_n \wedge |y - z| < 2^{-n}\}.$$

But the projections of Borel sets are A-sets.

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Continuing the proof of Lemma 1, we prove one more auxiliary lemma.

LEMMA 3. The graph $F = \{\langle x, f(x)\rangle : x \in OX\}$ of the function y = f(x) satisfies the equation $F = \bigcap_{n\in\omega} W_n$.

Proof. Let points x and y be such that y = f(x). We prove that $\langle x, y\rangle \in W_n$. By the choice of the sets E_n, for a given n there exists a point y' such that $\langle x, y'\rangle \in E_n$, and it satisfies the relation $|y' - f(x)| < 2^{-n}$. This means that $\langle x, y\rangle \in W_n$.

Conversely, let a pair $\langle x, y\rangle$ belong to all the sets W_n. We prove that y = f(x). Let us assume the contrary. Then there exists a natural number n such that $|y - f(x)| > 2^{-n+1}$. By the definition of W_n, we can choose a y' such that $\langle x, y'\rangle \in E_n$ and $|y - y'| < 2^{-n}$. But by the choice of the sets E_n, the inequality $|y' - f(x)| < 2^{-n}$ must be fulfilled. Now we have $|y - f(x)| < 2^{-n} + 2^{-n} = 2^{-n+1}$, which contradicts the choice of n.

This contradiction completes the proof of the equality y = f(x) and Lemma 3.

We return to the proof of Lemma 1. By Lemmas 2 and 3, the graph F of the function y = f(x) is the intersection of countable numbers of the A-sets W_n. By the same token, F is an A-set [3, p. 347]. But this contradicts the choice of the curve y = f(x), by which F cannot be an A-set. (Each set that belongs to both the classes A and CA is a Borel set by the Suslin theorem.)

This contradiction completes the proof of Lemma 1 and Theorem 1.

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LOCAL TORELLI THEOREM FOR BUNDLES ON MANIFOLDS WITH K = 0

K. I. Kii

Section 1

By the local Torelli theorem we shall mean the injectivity of the differential of the period map. The definition of the period map and the calculation of its differential are contained in Griffiths [1].

In Sec. 1 we shall prove the following theorem.

THEOREM 1. Let f: V → B be a surface with a bundle of elliptic curves with nontrivial functional invariant without multiple fibers and suppose |K| has no fixed components. Then the local Torelli theorem holds for V.

Proof. In [2] the author gives a method for verifying the local Torelli theorem for periods of n-forms.

We consider the commutative diagram:

$$\begin{array}{ccccccc} 0 \to H^0(V, \Omega^1(-K)) & \to & H^0(V, \Omega^1) & \xrightarrow{\pi_1} & H^0(K, \Omega^1|_K) & \to \\ \downarrow \omega_i & & \downarrow \omega_i & & \downarrow \omega_i & \\ 0 \to H^0(V, \Omega^1) & \to & H^0(V, \Omega^1(K)) & \xrightarrow{\pi_2} & H^0(K, \Omega^1(K)|_K) & \to . \end{array}$$

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