ISSN 0027-1322, Moscow University Mathematics Bulletin, 2025, Vol. 80, No. 1, pp. 23–33. © Allerton Press, Inc., 2025.
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Chair of Mathematical Logic and Theory of Algorithms

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Received July 21, 2024; revised January 12, 2025; accepted February 1, 2025

Abstract—The paper describes the history of the creation and activities of the Chair of Mathematical Logic and Theory of Algorithms at Lomonosov Moscow State University. A brief description of the areas of work and achievements of graduates of the chair and its fellows is given.

DOI: 10.3103/S0027132225700196

Keywords: *Chair of Mathematical Logic and Theory of Algorithms, Lomonosov Moscow State University*

1. INTRODUCTION

What mathematical logic and theory of algorithms do, what results in this field are associated with our chair at the Faculty of Mechanics and Mathematics of the Lomonosov Moscow State University (MSU)? The current work is an attempt of explaining this to specialists from the other branches of mathematics, and we hope that it will be also useful to students choosing the topic of their studies.

Speaking about the achievements of the chair, we follow not from the formal current (or past) affiliation of a researcher: many graduates and fellows of the chair also work at the HSE University,

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Moscow Institute of Physics and Technology (National Research University), research institutes of the Russian Academy of Sciences, and abroad. An exception would be the relations of the chair with the Steklov Mathematical Institute of the Russian Academy of Sciences, with which we begin our next survey. Almost all the mentioned mathematicians, except for obvious senior persons or those marked specially are graduates of the chair. We also do not provide first names and patronyms in full, do not provide the full formulations of the results, and often just limit ourselves by mentioning the names of the researchers and the field of their studies.

A more detailed survey will be presented at the website of the chair[16].

The heads of the Chair of Mathematical Logic and Theory of Algorithms include 1959–1979, Corresponding Member of the Academy of Sciences of Soviet Union A.A. Markov (1903–1979); 1980–1987, Academician A.N. Kolmogorov (1903–1987); 1987–1993, Academician V.A. Mel'nikov (1928–1993); 1993–2018, Professor V.A. Uspenskii (1930–2018); and since 2018, Academician A.L. Semenov.

2. MATHEMATICAL LOGIC AT THE LOMONOSOV MOSCOW STATE UNIVERSITY AND RUSSIAN ACADEMY OF SCIENCES

The Seminar on Mathematical Logic existed at the Faculty of Mechanics and Mathematics of MSU since 1943 under the guidance of P.S. Novikov (1901–1975), S.A. Yanovskaya (1896–1966), and I.I. Zhegalkin (1869–1947). In 1957, at the Steklov Mathematical Institute of the Academy of Sciences of Soviet Union (MIAS), the Department of Mathematical Logic was founded headed by P.S. Novikov, who made an extraordinary contribution to the set theory, the proof theory, the study of algorithmic problems in algebra, and to the combinatorial group theory. His student, later Academician of Russian Academy of Sciences (RAS), Professor of the chair S.I. Adyan (1931–2020) substituted him at the position of the head of the department, and the department is now headed by Academician of RAS L.D. Beklemishev.

As a rule, the fellows of the MIAS department are graduates of our chair and maintain permanent relationship with it.

3. MODELS OF THINKING AND COMMUNICATION

The mathematical logic and theory of algorithms constructs mathematical models of human thinking, communication, and information activity and is a fundamental basis of cognitive sciences. This explicit appearance of a human distinguishes our subject from the other branches of mathematics, leads to the use of humanitarian sources of problems and applications. The constructed models are applied to construct artificial intelligence systems. Professor V.A. Uspenskii (1930–2018) proposed an initiative of reforming the linguistic education by introducing a course of mathematics in it, creating the department of full-time language studies (the Department of Theoretical and Applied Linguistics) at the Philological Faculty of MSU. The fellows and graduate students of our chair teach mathematics there. The mathematical and programming courses of the department reflect the state-of-the-art of mathematics and its applications in digital technologies and artificial intelligence.

4. FOUNDATIONS OF MATHEMATICS AND COGNITIVE BOUNDARIES

The problematics of mathematical logic is equally ancient as arithmetic and geometry. In his *Elements*, Euclid attempted to describe exactly what is a geometric truth by prescribing the system of proof. Aristotle attempted to describe the universal logic of human thinking. The end of the 19th century was marked by the mathematical description of what is a formal mathematical language and by the exact description of what is the subject and method of proof in the mathematics — the theory of Cantor sets and calculus of relations (predicates).

The 20th century began with the International Mathematical Congress in Paris, where Hilbert formulated his famous problems, several of which belong to the mathematical logic. He also announced the program of proving the completeness and consistency of mathematics by reliable tools operating

[16] Chair of Mathematical Logic and Theory of Algorithms. http://logic.math.msu.ru.

only with finite objects — chains of symbols or natural numbers. Kurt Gödel showed that the Hilbert program cannot be implemented.

An alternative variant of constructing the foundations of mathematics was the Brauer intuitionism, which, in addition to the subject of mathematical reasoning, restricts its method of reasoning: it is assumed that a statement "A or B" is proved only if some of the alternatives is proved; the law of excluded middle is rejected. The constructivism became a relevant direction in the foundations of mathematics; in it the existence of some object is assumed equivalent to the possibility of its explicit computation, construction. The founder of the Russian constructivism was A.A. Markov.

By 1930s, in the mathematical logic and theory of algorithms, researchers have fully established the exact mathematical representations about what is provability, truthfulness, determinability, and computability.

The studies in the field of foundations of mathematics also led to the results about independence. The understanding of independence was the core of the Lobachevsky discovery: he proposed the independence of the parallel postulate from other Euclid's axioms and began to construct the non-Euclidean geometry. Taking a natural set of statements as axioms of the set theory, we can supplement these axioms by the axiom of choice or its negation, by the continuum hypothesis or its negation: any of these four supplements will not lead to a contradiction in the set theory if the original theory was consistent. The students of the V.A. Uspenskii, V.A. Lyubetsky and V.G. Kanovei perform studies in the field of axiomatic set theory [1, 2]. They solved the Tarsky problem about the minimal complexity of describing a set consisting of real numbers defined by formulas having complexity no higher than any fixed natural number, where the complexity is understood as the number of transitions to the power set.

5. THEORY OF PROOFS. INTUITIONISM, CONSTRUCTIVISM. LOGIC OF PROBLEMS

Beginning with Hilbert and Gödel, the mathematical logic develops the proof theory, which solves the questions about the relation of different systems to each other, about their equivalence or mutual interpretability.

A student of A.A. Markov, A.G. Dragalin (1941–1998) with his students develop, on the one side, the approach to the proof theory from point of view of intuitionism and constructivism and, on the other side, overstepping the purely finite methods of this theory. Dragalin developed many directions of the proof theory, for instance, the semantics of intuitionistic systems, by proposing the class of models generalizing the Kripke models [3].

The topics of studies of P.S. Novikov on the proof theory, axiomatic systems for arithmetic are continued by L.D. Beklemishev and his descendants, D.S. Shamkanov and F.N. Pakhomov. In particular, they investigate the provable properties of computable functions, as well as Turing progressions — the sequences of theories obtained by supplementing consistency statements. L.D. Beklemishev proposed to analyze the axiomatic theory based on structures of reflection schemes, constructed new unprovable statements of the combinatorial character, and obtained the finest known classification of the consequences of arithmetic theories by their quantor complexity [4, 5]. D.S. Shamkanov investigates non-well-founded and cyclic proof systems, where it is possible to refer both backward and forward in the proof, which is applied to languages with different types of induction and recursion [6].

The works of A.N. Kolmogorov of 1925 and 1932 [7, 8] are joined by a common idea: to bridge the gaps between the classical and intuitionistic understandings of mathematics. In 1932 he proposed an idea of joint proposition and problem logic. D.P. Skvortsov and V.B. Shekhtman dealt with implementation of this idea [9], and in recent years S.A. Melikhov (a graduate of the Chair of General Topology of MSU) work on this topic and A.A. Onoprienko [10].

The constructivism of A.A. Markov, based on the formal notion of algorithm, was developed by his descendants, A.G. Dragalin and S.N. Artemov. The studies of logics, where the ideas of constructivity and modality, provability are used, are continued by V.N. Krupski and V.E. Plisko, students of V.A. Uspenskii [11], as well as by T.L. Yavorskaya, a student of S.N. Artemov [12].

6. LOGIC OF POSSIBLE AND NECESSARY, OTHER MODALITIES. LOGIC OF PROVABILITY. LOGIC OF KNOWLEDGE REPRESENTATION

Trying to mathematically describe the most broad circle of human reasoning, mathematical logicians considered the modalities: probably, necessary, sometimes is, etc., and the corresponding *modal* logics. They are tightly related to intuitionistic logics. Professor of the chair V.B. Shekhtman has several classical results in this field, where also his students are active: I.B. Shapirovskii, A.V. Kudinov, S.P. Kikot' [13–18].

S.N. Artemov, L.D. Beklemishev, and their descendants consider logics with the provability assumption. In particular, Artemov and, independently, a student of Uspenskii, V.A. Vardanyan, proved the impossibility of axiomatization of predicate logic of provability [19, 20]. Furthermore, for several its fragments, Artemov et al. proved the solvability. Beklemishev obtained a classification theorem for propositional logics of provability, as well as the results on the completeness for polymodal logics of provability describing the joint behavior of several arithmetic theories [21, 22]. Studying the polymodal logic of provability is a rapidly evolving field, which has deep relations to the axiomatic set theory, ordinal analysis, topology, and knowledge representation logic (description logic).

S.N. Artemov proposed a *justification logic* [23–25], which includes statements of the form "x is a justification of A" and in which, in addition to logic operations with the statements, the operations with justifications are considered. He established the relations of these logics with logics formalizing the notion of knowledge in a multiagent system. T.L. Yavorskaya [12, 36] and V.N. Krupski [27, 28] work in the field of justification logic. L.D. Beklemishev together with Yu.Sh. Gurevich investigated the "infon logic," used in authentication languages [29]. The constructive knowledge logic based on the intuitionistic modal logic was introduced by S.N. Artemov, and the work by A.A. Onoprienko is tightly associated with the studies of this logic [30, 31]. V.A. Vardanyan studied the reflexive knowledge logic, in particular, formalizing the situation of "mudded wiser heads." The description logics used to represent knowledge were put as basis for the OWL computer language of network ontologies. E.E. Zolin is engaged in studying description and other modal logics.

7. LOGIC OF STRUCTURES. WHAT CAN BE DEFINED, WHAT CAN BE VERIFIED?

A.L. Semenov, together with a student of Uspenskii, S.F. Soprunov, examines the question of what can be defined, expressed in a formal mathematical language having some initial set of notions [32]. An early died remarkable mathematician An.A. Muchnik (1958–2007), a student of A.L. Semenov worked with them. An example of the result on the definability is the Cobham–Semenov theorem on definability through addition of natural numbers of all relations given by finite automata in two essentially different numeral systems [33]. The proof of this theorem discovered by Muchnik [34], based on the notion of self-definability dating back to Tarsky, have found many applications. Semenov and Soprunov described a definability lattice for integers with successor [35]. They also constructed examples of definability hierarchies by quantor depth [36] and by number of arguments of generating relations [37].

Among the other results of Semenov is the proof of solvability of the theory of structure of natural numbers with addition and an exponent operation with a fixed natural base [38]. In work [39] Muchnik solved the problem concerning the solvability of monadic (admitting quantors by subsets) theory of infinite tree posed by M. Rabin at the International Mathematical Congress in Nice. Soprunov solved the Elgot–Rabin problem by establishing the nonexistence of the maximal solvable monadic theory [40].

8. COMBINATORICS ON WORDS. ALGORITHMIC PROBLEMS

In 1930, the works by Gödel, Post, and Turing introduced the formal definition of computability of a function and solvability of the problem of constructing an algorithm, including the algorithm of recognizing a property of an object given by a word. At the end of 1940s, the definition of a computable function over words was proposed by A.A. Markov: even before the beginning of the digital era, he constructed tools of structural programming and inductive proof of program correctness, proved the correctness of operation of an interpreter (for his own — abstract — algorithms). The general form of the algorithms working with graphs was described by Kolmogorov and Uspenskii at the beginning of 1950s.

A.A. Markov, P.S. Novikov, and S.I. Adyan proved the algorithmic undecidability of important algorithmic problems: Markov, for the problem of equality of words in a finitely presented semigroup (the Thue problem); P.S. Novikov, the same for groups (the Dan problem); S.I. Adyan, for a broad class of properties of a group by its prescription by relations (the Adyan—Rabin theorem); based on that, A.A. Markov proved the undecidability of the problem of homeomorphy of topological manifolds in dimensions > 3.

8.1. Solution to Burnside Problem and Algebraic Applications

Even in 1902, W. Burnside posed the problem of finiteness of a group with a finite number of generators in which one and the same power of each element is unity. An extremely difficult solution to this problem was obtained by P.S. Novikov together with his student S.I. Adyan. The methods developed during this research allowed Adyan to solve a series of open problems in the group theory.

A.Ya. Belov and his students continue the studies on combinatorial algebra (combinatorics on words), including the studies with application of geometric methods. In particular, they have constructed a finitely presented semigroup in which one and the same (ninth) power of each element is zero [41], as well as the proof of algorithmic undecidability of the problem of embedding for algebraic varieties. Belov advanced in the Jacobian conjecture [42]. He proved the local finite basis property of identity systems for associative rings (for an infinite set of variables he constructed counterexamples) [43, 44]. For algebras over the field of characteristic 0 the finite basis property (that is, the solution to the Specht problem) was earlier obtained by A.R. Kemer.

8.2. Language Models. Mathematical Linguistics

Let us return from the languages of mathematics to the languages of human and their models in the mathematical logic.

The first works of A.L. Semenov were devoted to the field of algorithmic analysis of context-free grammars and structural properties of languages they generate [45]. M.R. Pentus and his students, S.L. Kuznetsov, A.A. Sorokin, and T.G. Pshenitsyn investigate the mathematical linguistics. They study categorial grammars, that is, calculi introduced by Lambek to model the syntax of natural language.

The key results about Lambek calculi were obtained by Pentus: the proof of the Chomsky conjecture about the equivalence of Lambek grammars and context-free grammars [46]; the completeness of the Lambek calculus with respect to language semantics [47]; the pseudo-polynomial algorithm for establishing the deducibility. Sorokin studied extensions of context-free grammars and Lambek grammars by “discontinuous” synactic operations [48, 49]. Kuznetsov, independently and with coauthors, obtained the results on the properties of different variants and extensions of the Lambek calculus, linear logic, and action logic [50]. Pshenitsyn examined a generalization of the Lambek grammars to hypergraphs [51]. M.E. Vishnikin, a student of Pentus and Kuznetsov, obtained the results concerning the basic categorial grammars.

8.3. Superwords. Symbolic Dynamics

Words and their infinite analogs, superwords, are investigated in the theory of dynamical systems: if the states of the system are recorded at discrete time instances with a limited accuracy, then a superword arises. In a wide class of situations this superword is almost periodic, that is, contains often repetitions of any word, which is met in it infinitely many times. Semenov [52], Muchnik, and their student Yu.L. Pritykin [53] dealt with the logic of superwords.

A student of A.Ya. Belov, I.V. Mitrofanov (and, independently of him, F. Durand, former President of the Société Mathématique de France) gave an answer to the question about algorithmic decidability of the problem of almost periodicity of HDOLL systems posed by Semenov [54, 55]. Ph.D. Rukhovich (a graduate of the Moscow Institute of Physics and Technology (National Research University)), a student of Belov and Semenov, investigated aperiodic trajectories of outer billiards by applying computer methods [56]. V.A. Timorin, A.Ya. Belov, and Ph.D. Rukhovich established the existence of an aperiodic trajectory for an outer billiard outside a regular n-gon for $n \neq 3, 4, 6$, thus solving the open question posed in his plenary report by R. Schwartz at the International Mathematical Congress in 2022.

9. THEORY OF ALGORITHMS

9.1. Computational Complexity. The P versus NP Problem

The P versus NP problem is the most important and mysterious problem of the modern mathematics; it consists in revealing the fact whether some naturally arising problems can be solved faster than just by exhaustive search over the variants. An exact formulation of this problem and the theorem associated with it were established simultaneously by the American mathematician S. Cook and a student of the chair, a descendant of A.N. Kolmogorov, L.A. Levin [57].

An impressive partial solution to this problem was obtained by student of the chair A.A. Razborov, who constructed algorithms of some natural form [58]. For this result he received the Nevanlinna Prize of the International Mathematical Union.

The topic of computational complexity is developed by the students of V.A. Uspenskii, N.K. Vereshchagin, A. Shen, and their descendants. The objects of study include one-sided functions, that is, functions that can be easily computed by hard to invert. They are applied in information security protocols, at construction of pseudo-random generators. The computational complexity is associated with the Kolmogorov complexity, communication complexity, and error-correcting codes [57].

9.2. Kolmogorov Complexity. Algorithmic Theory of Information

Kolmogorov has one of the most remarkable definitions of mathematics, the definition of complexity of an object as a minimal length of its description. The studies of complexity of finite objects were initiated by A.N. Kolmogorov in 1962. The problem he then formulated was solved 40 years after by An. A. Muchnik and A.L. Semenov [59, 60]. The survey of the first results on the algorithmic theory of information was made also by L.A. Levin (in collaboration with A.K. Zvonkin) in [61].

A wide spectrum of results on the theory of complexity was subsequently obtained by L.A. Levin, N.K. Vereshchagin, A. Shen, and An.A. Muchnik, and their students.

The approach of Kolmogorov is the basis of the modern understanding of the amount of information and measure of randomness. A.N. Kolmogorov considered a combinatorial, probabilistic, and algorithmic approaches to information and parallels between them [62]. Later, the metatheorems for such parallelism were obtained. For instance, the classes of linear inequalities for the Shannon complexity and entropy coincide (the Romashchenko theorem), and these inequalities have a natural interpretation in terms of sizes of sets, their parts, and projections (Vereshchagin) [63–69].

Muchnik established [70] that the statements of the general theory of computability can be considered to be winning statements in some game. In the works of Muchnik, Vereshchagin, and their students, the game technique was applied to the theory of Kolmogorov complexity [39]. In particular, Muchnik obtained a complete description of the possibility of extracting the general information from objects with a given reciprocal information [71, 72].

9.3. Algorithmic Statistics. Randomness

The last area of interests of A.N. Kolmogorov was the algorithmic statistics founded by him. The studies in this field are continued in the scientific school of V.A. Uspenskii, N.K. Vereshchagin, An.A. Muchnik, and A. Shen. The classical probability theory does not explain why, having a sequence of thousand zeros obtained by flipping a coin, we reject the conjecture that the coin was symmetric and the flips were independent. The theory of algorithms allows explaining this fact — the Kolmogorov complexity of this sequence is negligible. Kolmogorov defined the *degree of stochasticity* of a finite sequence. Graduates of the chair, students of V.A. Uspenskii, V.V. V'yugin and A. Shen managed to answer the question of Kolmogorov: they estimated the number of nonstochastic sequences. Student of Kolmogorov Swedish mathematician P. Martin-Löf provided the corresponding definitions for infinite sequences. Several theorems covering different approaches to the definition of randomness were presented in [73]; the theorems establishing the relation between the conditional complexity and codes were given in [74].

For an object x we can seek (1) its complexity with limitation on resources; (2) the set x containing it and in which x is a typical representative; (3) the Kolmogorov structural function, that is, the minimal size of the set of bounded complexity containing x. The equivalence (with some accuracy) of these approaches was established by Vereshchagin (in collaboration with P. Vitányi from the Netherlands) [75].

An.A. Muchnik and A.E. Romashchenko, as well as A.S. Milovanov, obtained the results on the properties of stochastic (having good models) and antistochastic (maximally badly modeled) objects.

9.4. Efficient Algorithms on Words and Graphs

V.A. Lyubetsky works in the field of theory of models and set theory, as well as in the field of bioinformatics, where algorithmic and computer models of genome and cellular processes are constructed, for instance, the models of tissue regeneration, development of the end brain, and formation of cell types in evolution and in individual development of organisms [76]. Using such models, a new gene responsible for tissue regeneration was found, and a database of cell types is constructed. M.A. Roitberg (1952–2017) and a student of Semenov, T.A. Starikovskaya dealt with algorithms of bioinformatics.

V.A. Lyubetsky, K.Yu. Gorbunov, O.A. Zverkov, et al. [77] investigate the problems of discrete optimization. Examples of their results include the following: For known operations over oriented weighted graphs consisting of chains and cycles, an algorithm is found with the linear execution time that transforms any two graphs one to another in the shortest way.

Construction of efficient algorithms on graphs is the topic of studies of Vereshchagin's student M.A. Babenko.

10. COMPUTER TECHNOLOGIES. ARTIFICIAL INTELLIGENCE. AUTOMATION OF PROOF

Since 1988 to his decease in 1993, our chair was headed by V.A. Mel'nikov, a participant of creating the most famous Soviet computer BESM-6 and other domestic computers, including supercomputers (A.L. Semenov also participated in the latter work).

Since the beginning of creating computers in 1950s, researchers attempted to use them to automatize the forms of human intelligent activity that were not fully described or lead to NP problems. Tremendous progress was achieved in this field, artificial intelligence. They are associated, in particular, with machine learning systems, in which computer itself constructs the algorithms, following from a large data array. V.V. Podol'skii investigates this research direction.

Appearance of a computer as a tool for automation of intelligent activity led to creation of computer tools of automation of constructing and verifying mathematical proofs. The students of the Faculty of Mechanics and Mathematics and other faculties and universities get acquainted with the most famous tool of that kind, the Coq system, in the course and workshop of V.N. Krupski and S.L. Kuznetsov.

11. MATHEMATICAL EDUCATION AT SCHOOLS

The humanitarian component of the mathematical logic and theory of algorithms discussed above and the content of this subject lead to the pedagogics, that is, to the questions of teaching mathematics.

A.N. Kolmogorov founded the Physical-Mathematical Boarding School at MSU (now, the Advanced Educational Scientific Center (Faculty) — Kolmogorov's Boarding School of Moscow State University), as well as the journal *Kvant*, where A.L. Semenov was the Editor-in-Chief in the 21st century.

A.L. Semenov was organizer and member of the author team of the first domestic textbook on informatics for all schools of our country, was rector of a pedagogical university, and now heads the Axel Berg Institute of Cybernetics and Educational Computing of the Federal Research Center Computer Science and Control of the Russian Academy of Sciences. During several decades, Semenov leads publication of textbooks on informatics for preparatory school, where the content is designed following from the modern understanding of the foundations of mathematics and informatics and is given in a visual form. His activity was awarded by the UNESCO Prize.

S.F. Soprunov is the leader of an important direction in the creative application of computer at school in teaching programming to children, which uses the Logo language and the educational philosophy of constructivism.

Graduates and members of the chair A.A. Onoprienko and S.L. Kuznetsov play the key role in the work of the Faculty of Mechanics and Mathematics with schoolgoers, including those of the Youth Faculty.

The fellows of the chair also participate in the work of project shifts for schoolgoers on the basis of the Educational Center Sirius: shifts on the definability theory, substitutive tesselations, and the Burnside problem.

A.Ya. Belov actively participated in the organization of the Moscow Mathematical Olympiad, was jury president of the International Internet Olympiad. At different times, he was the head of student teams at international mathematical olympiads, and for more than 30 years he was the jury president at the Sommer Conference of the Tournament of the Towns.

FUNDING

This work was supported by ongoing institutional funding. No additional grants to carry out or direct this particular research were obtained.

CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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Translated by E. Oborin