Fully saturated extensions of standard universe

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It seems that it has been taken for granted that there is no distinguished, definable, countably saturated nonstandard model of the reals. Of course $\mathbf{V} = \mathbf{L}$ implies the existence of such an extension (take the first one in the sense of the canonical well-ordering of $\mathbf{L}$), but the existence provably in **ZFC** was established quite recently in [1]. (Without Choice the existence of *any* elementary extension of the reals, containing an infinitely large integer, is not provable.) The existence of a definable fully saturated (that is κ-saturated for any cardinal κ) elementary extension of the whole set universe of **ZFC** is an even more challenging problem.

Theorem 1 *There exists, provably in* **ZFC**, *a definable fully saturated elementary extension of the whole set universe of* **ZFC**.

Such an extension can be viewed as an interpretation of bounded set theory **BST** in **ZFC**, such that the standard core of the interpretation coincides with the **ZFC** universe. (**BST** is an improved, foundations-friendly modification of internal set theory **IST**, in which every set is postulated to belong to a standard set.) Such an interpretation of **BST** was earlier obtained only on the base of Global Choice variants of **ZFC**. It is known that **IST** itself does not admit such an interpretation.

The proof of Theorem 1 consists of an Ord-long chain of consecutive iterated ultrapowers of the set universe of **ZFC**.

[1] V. Kanovei and S. Shelah, A definable nonstandard model of the reals, *JSL* 2004, 69, 1, 159–164.

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