22nd Workshop on Computer Algebra
in memory of Professor Vladimir Gerdt

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Book of Abstracts

Laboratory of Information Technologies, JINR, Dubna

CCAS

MSU (CMC faculty)

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Preface

The workshop is a traditional joint meeting of the participants of MSU (CMC faculty) & CCAS Seminar on Computer Algebra and JINR (the Laboratory of Information Technologies) Seminar on Computational and Applied Mathematics.

The purpose of the workshop is to present studies on topics of current interest in computer algebra and new developments in the field of symbolic and algebraic computations with a special attention to their different applications in mathematics, physics and engineering.

The Workshop will be dedicated to the memory of our dear friend and outstanding scientist Professor Vladimir Gerdt to commemorate his contributions to the field of symbolic and algebraic computations and their diverse applications to the problems of computational physics.

Being one of the leading experts in area of symbolic and algebraic computations, he was also significantly involved in the organisation of a scientific and educational activities, among other things, for many years he was a Co-Chairman of the Computer Algebra workshops.

Conference Chairmen,
S. A. Abramov (Computing Centre of RAS and MSU)
A. Khvedelidze (LIT JINR)

Acknowledgements

We would like to express our sincere thanks to everyone who helped us in preparing and organizing the Workshop at such a challenging time and in such a sophisticated format. We would especially like to mention the role of Ján Buša and Ján Buša Jr.

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Contributed Papers

Discovering nonexistence of solutions of linear ordinary difference and differential systems

Sergei A. Abramov[1], Denis E. Khmelnov[1] and Anna A. Ryabenko[1]

[1] *Dorodnicyn Computing Center Federal Research Center "Computer Science and Control" of the Russian Academy of Sciences, Vavilova 40, Moscow, Russia, 119333*

It is quite common that search algorithms for those solutions of difference and differential equations and systems that belong to a fixed class of functions are designed so that nonexistence of solutions of the desired type is detected only in the last stages of the algorithm. However, performing additional tests on the intermediate results makes it possible to stop the algorithm as soon as these tests imply that no solutions of the desired type exist. This gives an opportunity to save time and other computing resources. So, it makes sense to equip algorithms with checkpoints and some tests. We consider these questions in connection with the search for rational solutions of linear homogeneous difference and differential systems with polynomial coefficients, and propose a scheme equipped with such checkpoints and tests, and also report results of experiments with our implementation of the scheme in Maple.

The full description of the proposed algorithms can be found in authors' paper published in
ACM Communications in Computer Algebra, Volume 54, Issue 2, June 2020, pp 18–29; https://doi.org/10.1145/3427218.3427219

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Question about the algorithmization of the Hasse-Weil functions

Dmitry Abrarov[1]

[1] *NPO PKRV, Rostech, Moscow, Russia*

Special zeta functions arise as exact solutions of the kinematic part of the Euler-Poisson equations with the condition of their time-reversibility. These functions are represented by the Hasse-Weil functions of elliptic curves over the field of rational numbers Q. They have the meaning of an equivariant analytic continuation of the classical theta solutions. The Hasse-Weil functions have the structure of special L-functions $F = \sum_n^{\infty} a_n n^{-s}$, where the coefficients a_n are determined by the corresponding elliptic curves over Q. For the practical applicability of the above solutions, a natural question arises about the algorithmization of F. Does this efficient algorithm exist? If it exists, what is its structure? How does this relate to existing algorithms for L-functions and theta-functions? With quantum computing algorithms?

Acknowledgement. This work was supported by NPO PKRV

An explicit procedure for calculating the perimeter of an ellipse

Semjon Adlaj[1,2]

[1] *Department of Mathematics of the National University of Science and Technology "MISiS", Russia*
[2] *Federal Research Center "Informatics and Control" of the Russian Academy of Sciences, Russia*

We discuss a concise algorithm for calculating the perimeter of an ellipse, based on the concept of the generalized arithmetic-geometric mean (GAGM) which was shown to yield the modified arithmetic-geometric mean (MAGM) and the arithmetic-geometric mean (AGM) as two special cases.

Assume that β and γ are two positive numbers which squares sum to one: $\beta^2 + \gamma^2 = 1$. Put $\{x_0, y_0, z_0\} = \{\beta, 1/\beta, 0\}$, $r_0 = 1$, $\rho_1 = 1$, and recursively calculate

$$\{x_{n+1}, y_{n+1}, z_{n+1}\} = \left\{ \frac{x_n + y_n}{2}, z_n + r_n, z_n - r_n \right\}, \rho_{n+1} = \rho_n \frac{x_{n-1} - z_n}{x_n - z_n}, \tag{1}$$

where $r_n = \sqrt{2(x_n - z_n)\, r_{n-1}}$, then (for each positive integer n) the interval

$$\rho_n\,(x_n,\, x_{n-1}) \tag{2}$$

must contain the ratio $N(\beta^2)/M(\beta)$, where $M(x)$ is the arithmetic-geometric mean of x and 1, whereas $N(x)$ is the modified arithmetic-geometric mean of x and 1.

Note, as well, that the length is of the interval (2) is a product of two multiplicands: ρ_n and $(x_{n-1} - y_{n-1})/2$, the first of which converges to $\beta/M(\beta)$, whereas the second decreases (quadratically) to zero, as the (descending) sequence $\{x_n\}$ and the (ascending) sequence $\{y_n\}$ approach their (common) limit $N(\beta^2)/\beta$. So, denoting by ρ and x the respective limits of the sequences $\{\rho_n\}$ and $\{x_n\}$, we have

$$\frac{2}{\pi}\int_0^1 \sqrt{\frac{1-\gamma^2x^2}{1-x^2}}\; dx = \frac{N(\beta^2)}{M(\beta)} = \rho\, x.$$

We might also adopt the presented algorithm, along with the formula

$$\pi = \frac{M^2}{N-1},$$

where $M = M(\sqrt{2})$ and $N = N(2)$, in order to calculate the constant π, so we set

$$\{x_0,\, y_0,\, z_0\} = \{2,\, 1,\, 0\},\; r_0 = \sqrt{2},\; \rho_1 = 1$$

and apply recursive formulas (1) in order to successively calculate the intervals

$$\pi_n = \frac{2}{\rho_n^2}\left(\frac{1}{x_{n-1}-1},\, \frac{1}{x_n - 1}\right),$$

containing the constant π, where $\sqrt{2}/\rho_n$ converges to $M(\sqrt{2})$: $\pi_1 = (2, 4)$,

$$\pi_2 \approx (2.914213562373095049,\, 3.187672642712108627),$$

$$\pi_3 \approx (3.140579250522168248,\, 3.141680293297653294),$$

$$\pi_4 \approx (3.141592646213542282,\, 3.141592653895446496),$$

$$\pi_5 \approx (3.141592653589793238,\, 3.141592653589793238).$$

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Computation of the Euler-Poisson System Solutions in the Form of Formal Power Series by Power Geometry Algorithms

Alexander Aranson

The computation of solutions in the form of formal power series for the Euler-Poisson ODE system describing the motion of a rigid body around a fixed point is considered. For computations power geometry algorithms, modified by the author for programming and implemented in the form of CAS Maxima scripts and C++ programs, are used. Using these scripts and programs we calculate solutions of the Euler-Poisson system in the form of formal power series containing formal Laurent and Puiseux series with a nonzero finite principal part at the finite point of the independent variable. During calculating such series conditions arise for the parameters of the original equations - the expansibility conditions. Every known today condition for the general and local integrability of the Euler-Poisson system coincides with one of the computed expansibility conditions. Many series with the expansibility conditions coincided with the integrability conditions do not pass the Painlevet test. For two different series the expansibility condition coincides with the Grioli integrability condition. The relations between the power exponents of these two series coincide with the relations between the power exponents of an integer function of exponential type expansion and the power exponents of the Borel transformation of such entire function.

The calculated results show the correctness of the scripts and programs written by the author.

Reversibility of difference schemes for dynamimmcpl systems with a quadratic right-hand side

Ali Baddour[1], Mikhail Malykh[1,2], Oleg Kroytor[1] and Leonid Sevastianov[1,2]

[1] *Department of Applied Probability and Informatics, Peoples' Friendship University of Russia (RUDN), Moscow, Russia*
[2] *Laboratory of Information Technologies, JINR, Dubna, Russia*

According to the finite difference method, the differential equation

$$\frac{dx}{dt} = f(x), \quad f \in Q[x],$$

is replaced by an algebraic equation connecting the value of the solution of this equation at time t with the value at $t + \Delta t$. Let us call the first value the initial value and denote it as x, and the second one call the final value and denote it as $\hat{x}$. Following V.P. Gredt, we are interested in inheritance of algebraic properties of an exact solution by approximate ones.

We invastigate the difference analogue of the Painlevé problem: find all differential equations that can be approximated by algebraic equations defining birational correspondences between x and $\hat{x}$.

It is important for the study of mechanical models, because it is often noted that there must be a one-to-one correspondence between the initial and final data. Birational correspondence is a special case of one-to-one correspondence. When passing to difference schemes, any mapping becomes algebraic; therefore, reversibility is equivalent to the property under discussion.

Any system of differential equations with a quadratic right-hand side is approximated by a difference scheme defining a birational correspondence between the initial and final values as points of projective spaces (reversible difference schemes). To do this, it is necessary to replace the derivative with a finite difference, and the monomials $x_i x_j$ on the right-hand side with $\hat{x}_i x_j$.

For example, in the transition from a continuous model of $\wp$-oscillator to a discrete one, the algebraic integral of motion is not preserved and thanks to this fact we move from the group of birational transformations of an elliptic curve to the group of Cremona transformations. Thus the question should be raised not about the inheritance of the algebraic

properties of an exact solution by approximate ones, but about their transformation.

The results of computer experiments with these schemes are presented.

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Zeros of hypergeometric polynomials of many complex variables

Dmitry Bogdanov[1] and Timur Sadykov[2]

[1] *Moscow Center of Technological Modernization of Education, Moscow, Russia*
[2] *Plekhanov Russian University, Moscow, Russia*

In the talk, we will define a hypergeometric polynomial of several complex variables with a support in an arbitrary convex integer polytope P. In the case where the polytope P is Z^n-connected (that is, when any two integer points of P can be connected by a polygonal line with unit sides and integer vertices), this polynomial is uniquely defined up to a constant factor.

In the talk, we will consider the properties of such polynomials and, under certain assumptions, prove the optimality of their amoebas in the sense of Forsberg-Passare-Tsikh.

Algorithm for construction of PI-type fully symmetric quadrature rules on simplexes

G. Chuluunbaatar[1,2] O. Chuluunbaatar[1,3]
A.A. Gusev[1] S.I. Vinitsky[1,2]

[1] *Joint Institute for Nuclear Research, 141980 Dubna, Russia*
[2] *Peoples' Friendship University of Russia (RUDN), Moscow, Russia*
[3] *Institute of Mathematics and Digital Technology, Mongolian Academy of Sciences, 13330 Ulaanbaatar, Mongolia*

The fully symmetric quadrature rules up to 10 order with positive weights, and with points lying in the $d = 3, \ldots, 6$ dimensional simplex are constructed. The weights and abscissas are first calculated using Fortran program with an accuracy of 10^{-25} and refined using Maple program with an accuracy of 10^{-50}. In calculations we applied the modified Levenberg-Marquardt methods for solving systems of nonlinear equations with convex constraints.

The integrability condition in the normal form method

Victor F. Edneral[1,2]

[1] *Skobeltsyn Institute of Nuclear Physics of Lomonosov Moscow State University, Leninskie gory 1(2), Moscow, 119991, Russian Federation*
[2] *Peoples' Friendship University of Russia (RUDN), Moscow, Russia, 6 Miklukho-Maklaya st., Moscow, 117198, Russian Federation*

The purpose of this report is to demonstrate the search for integrability conditions by the normal form method. We have chosen an equation of the Liénard-type as the object of the demonstration. We presented the equation as a dynamical system and parameterized it. We constructed polynomial equations in the system parameters which should be satisfied for the integrable cases.

$$\dot{x} = y,$$
$$\dot{y} = (a_0 + a_2 x^2)\, y + d_0(1 + d_1 x)(1 + d_2 x)(1 + d_3 x)(1 + d_4 x).$$

Conditions for local integrability near stationary points are written. Some parameter values are found for which these conditions are fulfilled.

It is essential that the conditions of integrability are written for resonant cases. For resonance (1:1) we received at least one rational solution of the condition. It is

$$a_2 = -a_0 d_1^2, d_0 = 1, d_2 = -d_1, d_3 = 0, d_4 = 0.$$

At these values of parameters the system is integrable and has a form

$$\dot{x} = y,$$
$$\dot{y} = (1 + a_0 y)\left(1 - d_1^2 x^2\right)$$

and the corresponded integral of motion is

$$I(x, y) = \frac{a_0 y - \log(a_0 y + 1)}{a_0^2} + \frac{1}{3} d_1^2 x^3 - x.$$

This result was obtained using the MATHEMATICA system in a purely algebraic way.

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My friend Vladimir Gerdt. Memories of university years

Marina Frontasyeva[1]

[1] *Laboratory of Neutron Physics, JINR, Dubna, Russia*

Vladimir Gerdt is a graduate of Saratov State University where we studied in the group of theoretical and nuclear physics of the faculty of physics. That was the time when physics was the most fashionable area of education: "Only physicists are salt, all the rest are zero!" - It was the poetic slogan of the students. My memories of a four-year study in Saratov, and then MSci study in Dubna are presented and supplemented with black and white photographs from more than 50 years ago...

Generation of large random quasigroups

Alexei Galatenko[1], Anton Pankratiev[1] and Vladimir Staroverov[1]

[1] *Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, Moscow, GSP-1, 1 Leninskiye Gory, Main Building, Russia*

A finite quasigroup is a pair (Q, f), where Q is a finite set and $f\colon Q \times Q \to Q$ is invertibe in both variables. Cayley tables of finie quasigroups are Latin squares, and an arbitrary Latin square specifies a finite quasigroup.

Finite quasigroups are a promising structure for cryptographic applications. Shannon proved that substitution cipher based on Latin squares is perfectly secret. Several quasigroup algorithms took part in NIST cryptographic standard selection contests. The number of research papers on quasigroup-based cryptography is rapidly growing.

In some cryptographic applications, especially in case of public-key algorithms, it is highly desirable to be able to generate large families of large quasigroups. A possible approach to sovling this problem is to use objects that are "random" in some sense, since random quasigroups are known to possess a number of beneficial properties. Jacobson and Matthews proposed an elegant MCMC-based method for generation of

uniform distribution on the set of all quasigroups of the given order. However in case of large quasigroup orders it is desirable to have faster solutions (perhaps at the cost of worse "uniformity"). We propose two alternatives, namely the procedure based on random permutations and the procedure based on proper families of functions.

Consider the following algorithm that takes quasigroup order m as the input and generates a quasigroup of the order m as the output:

1. start from the empty Latin rectangle of the length m;
2. iteratively add random permutations and correct the rectangle so that Latin property is satisfied;
3. stop when a Latin square is obtained.

The correction procedure has the complexity $O\left(m^2\right)$, so the following asserion holds.

Theorem 1. *Any quasigroup of the order m is produced by the algorithm with non-zero probability. The complexity of the algorithm is $O\left(m^3\right)$.*

Our experiments showed that quasigroups of the order up to several thousand can be generated in a reasonable time, so the bottleneck here is memory required to store the Cayley table. The implementation of the algorithm is available at http://stargeo.ru/2d.zip.

The second approach works with multivariate representation of quasigroups. Assume that $|Q| = k^n$ for some natural k, n. Then elements of Q can be considered as n-tuples, and f can be treated as vector functions in $2n$ variables. A family of n-ary functions $(g_1, \ldots, g_n)$ is said to be proper if for any two n-tuples s, t there exists an index i such that $s_i \neq t_i$, but $g_i(s) = g_i(t)$. It is known that the representation

$$f_i(x_1, \ldots, x_n, y_1, \ldots, y_n) = x_i + y_i + g_i(\pi_1(x_1, y_1), \ldots, \pi_n(x_n, y_n))$$

specifies a quasigroup for arbitrary functions $\pi_1, \ldots, \pi_n$ if and only if the family $(g_1, \ldots, g_n)$ is proper. Thus a single proper family allows one to specify up to $\left(k^{k^2}\right)^n$ quasigroups. The advantages of this approach include reduced memory requirements ($n \cdot k^n$ elements for the functions g_i and $n \cdot k^2$ elements for π_i instead of k^{2n} elements of the Cayley table) and generation speed; the disadvantage is inability to generate all quasigroups. Randomness is achieved by variation of the functions π_i for a fixed proper family. The additional possibility is to use a random proper family generated by an MCMC procedure.

Solving some geometric algebra problems using the Grassmann.jl package for Julia language

Migran N. Gevorkyan[1], Dmitry S. Kulyabov[1,2], Anna V. Korolkova[1], Anastasia V. Demidova[1] and Tatiana R. Velieva[1,3]

[1] *Peoples' Friendship University of Russia (RUDN), Moscow, Russia, 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation*
[2] *Joint Institute for Nuclear Research, 6 Joliot-Curie, Dubna, Moscow region, 141980, Russian Federation*
[2] *Plekhanov Russian University of Economics, Moscow, Russia, 36 Stremyanny lane, Moscow, 117997, Russian Federation*

Geometric algebra is based on the works of Grassman and Clifford. The main entities studied are p-vectors (polyvectors) and multivectors. Polyvectors together with the external multiplication operation give the implementation of the Grassmann algebra and multivectors with the geometric multiplication operation implement the Clifford algebra. Multivector algebra allows one to generalize many operations and entities from analytical and differential geometries, such as vector and mixed products, normal and binormal vectors, etc., to the multidimensional case and give them a visual geometric interpretation. Complex numbers and quaternions are isomorphic to multivectors of a special kind.

In this paper, we will consider the application of the ideas of geometric algebra to solve some of the applied problems that occur in computer geometry. To solve these problems, the Grassmann package is used.jl for the Julia language.

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On the algorithm for computation of partial indices of a piecewise constant matrix function

Grigori Giorgadze

[1]*Department of Mathematics, Faculty of exact and Natural sciences of the*
[1]*Tbilisi State University, 0160 Tbilisi, Georgia*

The classical setting of the Riemann-Hilbert boundary value problem is to find piecewise holomorphic vector function $\Phi(t)$ in $U_+ \cup U_-$, which admits continuous boundary values on Γ, satisfies the boundary condition

$$\Phi^+(t) = f(t)\Phi^-(t), t \in \Gamma \tag{3}$$

and has finite order at ∞. Here Γ is a smooth closed positively oriented curve in the Riemann sphere CP^1, which divides CP^1 into two domains $0 \in U_+$ and $\infty \in U_-$, and $f : \Gamma \to GL_n(C)$ is a given Hölder continuous matrix function. We call f thetransition function of the boundary problem (3).

This problem is traditionally solved using the method of singular integral equations. Unlike of 1-dimensional case, where the problem is solvable in closed form, for n-dimensional case the existence of the solution is all that we know. We use the algebraic approach to solve the above boundary value problem and extend this technique for solving the problem in closed form in the case of piecewise constant transition function $f(t)$. Suppose $s_1, ..., s_m$ is a collection of points in the finite complex plane and $M_1, ..., M_m$ are non-degenerate matrices. Let us connect them by a simple closed contour Γ, i.e $s_1, ..., s_m \in \Gamma$ and Γ is a union of smooth non-intersecting arcs $\Gamma_1, ..., \Gamma_m$ with fixed orientation. Let the non-degenerate piecewise constant matrix function $f(t)$ be constructed from the collections $S = \{s_1, ..., s_m\}$, $M = \{M_1, ..., M_m\}$ in the following manner:

$$f(t) = M_j \cdot ... \cdot M_1, \ \ if \ \ t \in \Gamma_j, \tag{4}$$

where M_j are non-degenerate matrices.

Theorem. ([1]) Let piecewise constant transition function $f(t)$ of the boundary value problem (3) be given by (4) and admit Φ-factorization in L^p, for some p. Then boundary value problem (3) is solvable in closed form.

According to this theorem it is natural to call the *partial indices of piecewise constant matrix function* the splitting type of canonical vector bundles induced from the collections $S = \{s_1, ..., s_m\}$,

$M = \{M_1, ..., M_m\}$. By the solution of Riemann-Hilbert boundary value problem in closed form in our setting, we mean the computation of the partial indices of the corresponding non-degenerate piecewise constant matrix function as in the case of boundary value problem with Hölder continuous transition function.

We reduce the problem to computation of partial indices of a rational matrix function and after such reduction we use the algorithm of factorization of the rational matrix function by rank of moment matrices [2].

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Investigation of the Real Solutions of Algebraic Systems with Parameters Using Groebner Bases Construction Methods

Sergey A. Gutnik[1,2]

[1] *Department of Mathematics Econometrics and IT, Moscow State Institute of International Relations (MGIMO University), 76, Prospekt Vernadskogo, Moscow, 119454, Russia*
[2] *Moscow Institute of Physics and Technology, Institutskii per. 9, Dolgoprudny, 141701 Russia*

Gröbner bases construction methods were used to find the real roots of an algebraic system that determines the equilibrium orientations for two connected bodies, that moves along a circular orbit under the action of gravitational torque. To determine the real roots of the system of 12 algebraic equations the coefficients of which depend on 4 parameters, this system was decomposed using modern algorithms for Gröbner basis construction. The number of the equilibrium solutions was found by analyzing the real roots of the algebraic equations from the calculated

Gröbner basis. Evolution of the conditions for solutions existence in the dependence of the parameters of the problem was investigated. The effectiveness of the algorithms for Gröbner basis construction was analyzed depending on the number of parameters for the problem under consideration.

We estimated the size of the problem and the size of the computational resources at which the task can be solved using Gröbner basis construction method.

On (computer) algebraic aspects of p-adic cohomology theories

Nikolaj Glazunov[1]

Department of Nonsmooth Optimization,Glushkov Institute of Cybernetics, Kiev, Glushkov Prospect 40, Ukraine

After the author's presentation at the Computer Algebra and its Applications to Physics conference (Dubna, 2001), V.P. Gerdt invited the author to prepare an article for the conference collection. Such an article was prepared and published by the author: N.M. Glazunov. On algebraic geometric and computer algebra aspects of mirror symmetry. *Proc. of the Int. Conf. "Computer Algebra and its Applications to Physics."Dubna-2001.* Joint Institute of Nuclear Research, Dubna (2002) 104-113.

Recently Bhatt, Hesselholt, Kaledin, Kontsevich, Mattew, Petrov, Scholze, Vologodsky have obtained fundamental results in noncommutative cohomology. Just to understand the results of these authors one needs deep knowledge from algebra and topology. The aim of this note is to present some of (computer) algebraic aspects of these results in the framework of rigid and p-adic cohomologies. We do it in the direction of cohomology theories with an emphasis on formal structures over fields and rings of finite characteristic and over p-adic fields and rings and their extensions. Recall that formal schemes are used in rigid cohomology. More generally, de Rham-Witt cohomology, Hochschild-Witt, periodic cyclic homology, p-adic cohomology use such formal structures, in particular, Witt vectors. Also we present for purposes of our considerations formal groups, Dieudonne modules, log structures, Gauss-Manin connections, Picard-Fuchs equations , Frobenius structures. Examples are included.

Software for structural identifiability analysis: role of Gröbner bases and characteristic sets

Ilia Ilmer[1], Alexey Ovchinnikov[1,2,3] and Gleb Pogudin[4]

[1]*Ph.D. Program in Computer Science, CUNY Graduate Center, New York, USA*
[2]*Department of Mathematics, CUNY Queens College*
[3]*Ph.D. Program in Mathematics, CUNY Graduate Center, New York, USA*
[4]*LIX, CNRS, École Polytechnique, Institute Polytechnique de Paris, France*

In modeling of physical and biological phenomena with ordinary differential equations (ODE), it is important to know which parameters can be identified from the experimental data [1]. Structural identifiability is a theoretical property of the ODE model that can assess such possibility before actual experiments. We say that a parameter is structurally *globally* (*locally*) identifiable if it can be determined during the experiment from the input and output data uniquely (up to finitely many values). If there are infinitely many possible values, then the parameter is non-identifiable. In the latter case, one may be interested in finding identifiable functions, or combinations, of such non-identifiable parameters to mitigate the issue of non-identifiability.

In this work, we present a web-based program that allows to quickly assess identifiability of individual parameters and initial conditions as well as that of all parameter functions identifiable from single or multiple experiments. The structural identifiability toolbox is available at `https://maple.cloud/app/6509768948056064/`. This tool is based on three main algorithms:

1. An algorithm for assessing global and local identifiability of individual parameters and initial conditions, in which the global identifiability part relies on Gröbner basis computation [5, 6].

2. An algorithm for finding all globally multi-experiment identifiable combinations of parameters [7], which is based on characteristic set decomposition of radical differential ideals (in the current version, via the Rosenfeld-Gröbner algorithm [3]).

3. An algorithm for finding all globally single-experiment identifiable combinations of parameters, which is also based on characteristic set decomposition of radical differential ideals and, additionally, on an algorithm for computing generators of intersections of fields [7].

We will present the web app, show how these algorithms are combined together to improve the performance, and discuss how one could incorporate differential Thomas decomposition [2, 4] instead of Rosenfeld-Gröbner to improve the efficiency.

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Berezin integral and Berezinian: from exterior powers and recurrence sequences to identities in the Grothendieck ring of the general linear supergroup

Hovhannes Khudaverdian

University of Manchester, Manchester, UK; IPPI

We give definition of Berezin integral and discuss some examples. Then we study the Berezinians of linear operators in a superspace and in particular the characteristic function $R_A(z) = \mathrm{Ber}(1 + zA)$, where z is a complex variable. Our principal tool is the two power expansions of $R_A(z)$, at zero and at infinity. We also study a similar rational function taking values in a Grothendieck ring. Using these methods we come to the new formulas.

Quantum Mereology Based on Finite Quantum Mechanics

Vladimir Kornyak

Laboratory of Information Technologies, JINR, Dubna, Russia

Mereology is the study of the part-to-whole and part-to-part relations within the whole. In *quantum mereology*, the whole is an isolated quantum system ("the universe") in a given pure state, undergoing a given unitary (Schrödinger) evolution. Quantum mereology studies the interrelations between singled out subsystems of the universe ("observable system", "observer", "environment", etc.), the emergence of geometry from quantum entanglement, and other fundamental issues of quantum mechanics.
Schematically, our research boils down to the following:
An isolated quantum system, constructed in the framework of finite quantum mechanics, is decomposed into a tensor product of subsystems. By reducing the "universe" quantum state, we obtain mixed states for subsystems. This allows us to study energy interactions and quantum correlations between subsystems and their time evolution. For the corresponding computations, involving rather cumbersome combinatorics, we develop and implement algorithms based on computer algebra techniques.

Using Symbolic Computation in the Description Problem for Degenerate Homogeneous Hypersurfaces in $\mathbb{C}^4$

A.V. Loboda[1] and V.I. Sukovykh[1]

[1] *Voronezh State Technical University*

Big family of examples of Levi degenerate homogeneous tubular surfaces in the space $\mathbb{C}^4$ (see [1]) can be obtained by considering degenerate affine homogeneous surfaces

$$\{\Phi = 0\}, \text{ where } \Phi = -x_4+(x_1^2+x_2^2)+F_3(x_1,x_2,x_3)+F_4(x_1,x_2,x_3)+... \tag{1}$$

from $\mathbb{R}^4$ as their bases.

The 3-dimensional Lie algebra of affine vector fields ensuring the homogeneity of such a surface can be given by three basic (5×5)-matrices (with the last zero row). Tangency conditions for such fields on the surface (1)

$$(E_k(\Phi))_{|\Phi=0} = 0 \ (k = 1, 2, 3)$$

mean that the elements of the matrices are connected by an infinite system of bilinear relations with the Taylor coefficients of equation (1). Using the refined canonical form of equation (1), we can talk about $39 = 13 \times 3$ unknown elements of the discussed triple of matrices.

As the main tool for determining all these elements we use the requirement of closure of the linear span of three matrices according to the commutator $[E_i, E_j] = E_iE_j - E_jE_i$. In several cases, the corresponding system of quadratic equations was solved using Groebner bases. For example, the following Levi degenerate tubes over affinely homogeneous surfaces in $\mathbb{R}^4$ were received:

$$x_4x_3 + x_1^2 = x_3(1 + x_2)^\alpha, \ \alpha \in \mathbb{R} \setminus \{0, 1\},$$

$$x_4x_3 + x_1^2 = x_3 \ln(1 + x_2), \quad x_4x_3 + x_1^2 = x_3 \exp(x_2).$$

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Verification of calculations special functions in CAS using the Frobenius method

Ksaverii Malyshev[1] and Mikhail Malykh[2]

[1] *Skobeltsyn Institute of Nuclear Physics, M.V. Lomonosov MSU, 119991, Moscow, GSP-1, 1-2 Leninskiye Gory, Russia*
[2] *Peoples' Friendship University of Russia, 117198, Moscow, Miklukho-Maklaya str.6, Russia*

The scattering of plane monochromatic electromagnetic wave on dielectric ball filled by optically inhomogeneous matter is considered. CAS Maple is able to solve analytically linear ordinary differential equations for coefficients of the field in several cases, including the Lüneburg lens. In this last case Maple return the expressions in Whittaker and Heun functions. However, there are difficulties at work with these functions in Maple: sometimes the values of the functions are very large, their graphs have strange jumps and structures which are typical for graphs found with big rounding errors. In our talk we compare the results found in Maple and the results that we got using Frobenius series in CAS Sage.

Fractality of the quantum phase space of instantaneous heart rhytm and symbolic-numerical calculations in the Maple system

Mikheev S.A., Tsvetkov V.P. and Tsvetkov I.V.

[1] *Tver State University, 170100, Russia, Tver, Zhelyabova, 33*

The quantum phase space of instantaneous heart rhytm (IHR) Sq most fully and adequately describes the properties of cardiac rhythms and is built on the basis of large data of daily Holter monitoring (HM).

An essential element of this process is the procedure for quantizing the frequency and rate of change of the IHR, built and implemented based on symbolic-numerical calculations in the Maple system.

3D-visualization of S_q allows presenting in a compact form big data of the 24-hour HM. One of the important properties of S_q is its fractality. We carried out the proof of this property based on the 24-hour HM data of patients of the Department of Functional Diagnostics of the Tver Regional Clinical Hospital and the complex of programs compiled by the authors using symbolic-numerical calculations in the Maple system.

The estimates obtained showed that the S_q structure deviates from the fractal one by no more than 1%.

On Structure of Caustics in Dynamical Systems with Two Degrees of Freedom

Aleksandr Mylläri[1], Tatiana Mylläri[1], Victor Edneral[2,3] and Nikolay Vassiliev[4,5]

[1] *Dept. of Computers & Technology, SAS, St.George's University, St.George's, Grenada, West Indies*
[2] *Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Leninskie Gory 1(2), Moscow, 119991, Russia*
[3] *Peoples' Friendship University of Russia (RUDN), Moscow, Russia), 6 Miklukho-Maklaya street, Moscow, 117198, Russia*
[4] *Steklov Institute of Mathematics of the Russian Academy of Sciences, St. Petersburg, Russia*
[5] *Saint Petersburg Electrotechnical University ul. Professora Popova 5, 197376 St. Petersburg, Russia*

We study the structure of caustics in dynamical systems with two degrees of freedom. Generalized Hénon-Heiles model is used for experiments. Results of numerical integration are compared with those obtained analytically by using a formal Gustavson-like integral of motion. We use program LINA developed at JINR to construct a formal integral. This formal integral is used together with the hamiltonian of the system to study semi-analytically the evolution of caustics with changes of initial conditions and/or parameters of the model.

Lattice Field Theory on a Digital Quantum Computer

Yuri Palii[1,2]

[1] *Laboratory of Quantum Photonics, Institute of Applied Physics, str. Academiei 5, Chisinau, Republic of Moldova, MD-2028*
[2] *Laboratory of Information Technologies, JINR, Dubna, Russia, str. Joliot-Curie 6, Dubna, Russia 141980*

Main issues of modeling of a lattice quantum field theory on a digital quantum computer are reviewed. The representation of a fermion Hamiltonian in terms of the Pauli matrices via Jordan-Wigner transformation allows to model the Hamiltonian dynamics using a quantum circuit. The tensor network ansatz for a wave function borrowed from the quantum many body theory is an efficient method to deal with the huge Hilbert space and take in to account entanglement properties of a system. The results of calculations for the Schwinger model on few qubit quantum computers are represented.

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Model-oriented approach to syntactically controlled information-measuring system based on category theory

Viktor Rikhvitsky[1]

[1] *Laboratory of Information Technologies, JINR, Dubna, Russia*

In the 1980s, an information-measuring system for processing film information worked at the LCTA JINR for bubble chambers. Measurements by an interactive system was assumed decision-making by human operators, whom were given the opportunity to refuse measurements, to choose the course of measurements in the process, but nevertheless, there there was a certain model of the object of measurement, which made it possible to describe this activity in a non-procedural way, which at the same time allowed to give the process procedural semantics.

In the process of measurement, an information object was created with a structural description. We can assume that the description was in functional programming style. The streams of information had a syntactic description, that can be considered in the style of D. Knuth's attribute grammars. It can be considered, that it was syntactically driven measurement organization process. We can mean that these were attribute DCG grammars.

The theory of category is the algebraic apparatus that can capture the structures of syntax and semantics. and syntactically driven parsing and so it can become the basis for the theory of such systems.

The syntactically controlled measurement process was implemented not only in our laboratory, but also in other (ITEP, Protvino) and in other ways.

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A Gap Between Small and Large Systems of Linear Equations

Alexandr Seliverstov

Laboratory 6, Institute for Information Transmission Problems of the Russian Academy of Sciences (Kharkevich Institute), Bolshoy Karetny per. 19, build. 1, Moscow 127051, Russia

The multidimensional $(0,1)$-knapsack problem is a well-known optimization problem. In 1994 Nikolai Kuzyurin discovered an average-case polynomial-time algorithm to solve it under some conditions. His approach is based on binomial tail bounds. Let us consider the corresponding decision problem whether there exists a binary solution to a system of inhomogeneous linear equations with integer coefficients. An algebraic approach allows to specify the structure of the set of inconvenient inputs. For example, if both number of unknowns n and number of equations m satisfy the inequality $2n \geq (n-m+1)(n-m+2)$, then the set of inconvenient inputs is included in the set of zeros of an explicit nonzero multivariate polynomial. Such systems are large because m is close to n.

On the other hand, this decision problem can be reduced to its particular case when the system consists of just one linear equation. It is known as the set partition problem. In some cases, the equation has small integer coefficients. Furthermore, a binary solution to one linear equation can be found using a pseudopolynomial-time algorithm. In the general case, the equation has large coefficients.

It is easy to show that the existence of a binary solution to one linear equation in n variables with bounded coefficients is equivalent to the existence of a binary solution to a system of $O(n)$ linear equations in $O(n \log n)$ variables. This system can be chosen in many ways. The simple proof is based on the Chinese Remainder Theorem. Note that we work over the ring of integers, but not over any field.

So, there exists a gap between two known cases. If the number of equations m satisfies the inequality of the type $m \geq n - n^{1-\varepsilon}$, then generic-case complexity is polynomial. On the other hand, if the number of equations is sufficiently small $m = O\left(\frac{n}{\log n}\right)$, then the decision problem is as hard as in case of one equation, i.e., when $m = 1$.

On super Plücker embedding and cluster algebras

Ekaterina Shemyakova[1]

[1] *Department of Mathematics and Statistics, University of Toledo, Ohio, USA*

The talk is about a new result: a super analog of the classical Plücker embedding of the Grassmannian into a projective space. Besides the general interest of such a construction, we are also motivated by the hope of constructing the much sought-after notion of super cluster algebras.

I will start with the notion of super exterior powers and why they cannot be introduced straightforwardly generalizing the classical case. I will explain the difficulties with identifying the target space for the desired super Plücker embedding, and then I would outline ideas of our solution to this problem.

It is known that Plücker relations in the classical case cerve as one of the prototypal examples for cluster algebras. Super Plücker relations that we obtained in the simplest case show some "super cluster structure". Time permitting, I will speak about examples such as super Grassmannians $G_2(\mathbb{R}^{4|1})$ and $G_2(\mathbb{R}^{5|1})$.

Based on arXiv:1906.12011 [math.DG].

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Features of Some of Our Computer Algebra Applications

Margarita Spiridonova, Stoyan Poryazov, Emilia Saranova and Velin Andonov

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Acad. G. Bonchev Str., bl. 8, 1113 Sofia, Bulgaria

The beginning of our research in the field of Computer Algebra and the features of our further research on some Computer Algebra (CA) topics are presented. They are considered in parallel with our collaboration with JINR Laboratory of Information Technologies, which started more than 30 years ago and was strongly supported by V.P. Gerdt.

The following topics are briefly considered:

- Use of the built-in capabilities of the general purpose Computer Algebra Systems (CAS) Reduce, SAC-2, Maple and Mathematica for problem solving.
- Development of special purpose CAS and program packages.
- Development of some conceptual and methodological problems.
- Non-standard and perspective applications (e.g. topical issues in quality of service prediction in the telecommunication networks).

Notes on our participation in some national and international activities are also included.

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On a property of the Knuth equivalence classes of permutations and the Schutzenberger involution

Nikolay Vassiliev[1]

[1] *St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, 27 Fontanka, St. Petersburg, Russia*

The remarkable Robinson Schoensted Knuth correspondence RSK plays an important role in algebra, combinatorics, asymptotic representation theory, and dynamical systems theory. RSK algorithm establishes a bijection between sequences of elements of linearly ordered set and the pairs of Young tableaux of the same shape called insertion tableau P and recording tableau Q. This correspondence induces the so-called Knuth equivalence relation on the symmetric group. Two permutations or two input sequences of a linearly ordered set are equivalent if their insertion tableaux P are coincide. We prove a theorem that in each Knuth equivalence class, the first k elements of the input sequence are uniquely determined by the last k elements of the Schutzenberger involutions of their recording tableau Q. For the case $k = 1$, this theorem proves the hypothesis formulated earlier by N.Vassiliev, V.Duzhin and A.Kuzmin that the first elements of Knuth-equivalent permutations are uniquely determined by the ends of the Schutzenberger nerves of their numbering tableau. The proof is based on the study of the dynamics of the tableaux P and Q under the action of the RSK transformation and the connection of this dynamics with the Schutzenberger involution. This result gives us an algorithm for calculating the first k elements of the permutation using the insertion tableau P and the last k elements of the Schutzenberger involution of the recording tableau Q.

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Distributed Parallel Computation of Involutive and Gröbner Bases Using the Tableau Representation of Polynomials

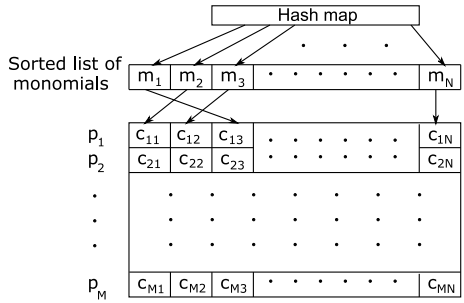
Denis A. Yanovich[1]

[1] *Laboratory of Information Technologies, JINR, Dubna, Russia*

In this work an attempt for using new tableau representation of polynomials for distributed parallel computing involutive and Gröbner bases of systems of nonlinear polynomial algebraic equations was made.

The computation of Gröbner (and involutive) bases is a challenging computational task both from the viewpoint of memory usage and computation time. It is also known that the complexity can be reduced computing the bases with the coefficients from the ring of modular numbers (or Farey numbers) and then reconstructing the basis with coefficients from Q.

The set of polynomials represented by a unified tableau with the columns assigned to the monomials that are currently included in the basis and the rows correspond to the polynomials. The tableau cells contains the coefficients of monomials. To ensure the random access to the coefficients of the polynomial by its monomials and to simplify the work with data, we add to this tableau some more structures: the linked list of monomials arranged in descending order of $\prec$ each element of which is assigned the index of its column in the tableau; the hash table given the key-monomial produces the index of its column in the tableau and the hash table given the column index in the tableau produces the corresponding monomial.



Usually, parallelization of a program is expected to speed up the computation time, but this work will consider the aspect of saving computing resources. Since the main structure in this view is a two-dimensional matrix, the simplest approach to sharing resources across

a network of machines is to allocate parts of the matrix for placement on different nodes.

Calculations of reductions of polynomials are performed simultaneously at all nodes, then the leading monomial is chosen by voting.

Using proposed structure, several realizations of this approach was made. One using MPI parallelization technology at the cost of average 2x slowdown shows almoust linear reduction of memory needed for computations. Another realizations was made using GRID like scheme of computations. The command node is using embedded C Civete webserver to distribute workload to cloud of the connected clients. Each client have the portions of the tableau and the command node have the full tableau in memory mapped file. The command node and clients communicates with the help of websockets.

Acknowledgement. This work is dedicated to the memory of my teacher, scientific advisor and brilliant mind V.P.Gerdt, who died suddenly this year. He will be remembered.

Investigation of Magnet for Spin Particle Detector at NICA-project

Ivan P. Yudin

Joint Institute for Nuclear Research, Dubna, Russia

The choice of magnet system for the Spin Physics Detector of the NICA Collider of LHEP JINR is given. The inverse problem of magnetostatics is solved for a magnetic field of 0.5 tesla in the aperture a) 3 m x 5 m and b) 3 m x 6 m. We also discuss the design of the magnet with a field of 0.3 T.

The paper presents the results obtained for the "warm" and SC versions of the magnetic system: currents (ampere-turns), the geometry (size) of the coil and the iron yoke, weight (on the whole and the individual elements), the magnet transportation and assembly.

Numerical method for round-error growing estimation.

M.I. Zuev and S.I. Serdyukova

Laboratory of Information Technologies, JINR, Dubna, Russia

In numerical calculating the current-voltage characteristics of Josephson junctions the Runge-Kutta method of the fourth - order accuracy is usually used. The calculations are performed at large time intervals, and at each time step the right-hand sides of the equations are recalculated four times. To shorten the calculating time, using ëxplicit" scheme of the second order accuracy has been suggested instead of the Runge-Kutta method. In case of $\tau = h$, estimates of $\|G^n\|$, guaranting boundedness of round-error grows for all n, have been proved, G is the operator of trasition from layer to layer; τ, h are the grid step sizes in t and x respectively. In this work a numerical-analitical algorithm for round-error growing estimation is developed for all $\tau \leq h$. Their boundedness was stated on the whole interval of IVC calculating long Josephson junctions using the suggested scheme. The calculations were performed on the supercomputer GOVORUN (LIT JINR) using the REDUCE system.