▶ VLADIMIR KANOVEI, *Some applications of finite-support products of Jensen's minimal Δ^1_3 forcing.*
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Jensen [4] introduced a forcing notion $P \in \mathrm{L}$ such that any P-generic real a over L has minimal L-degree, is Δ^1_3 in $\mathrm{L}[a]$, and is the only P-generic real in $\mathrm{L}[a]$. Further applications of this forcing include iterations, finite products and finite-support infinite products for symmetric choiceless models [1], et cetera. We present some new applications of finite-support infinite products of Jensen's forcing and its variations.

THEOREM 1 ([5]). *There is a generic extension* $\mathrm{L}[x]$ *of* L *by a real* x *in which* $[x]_{\mathsf{E}_0}$ *is a (lightface)* Π^1_2 *set containing no OD (ordinal-definable) reals. Therefore it is consistent with* **ZFC** *that there is a* **countable** *non-empty lightface* Π^1_2 *set of reals, in fact a* E_0 *equialence class, containing no OD elements.*

Recall that E_0 is an equivalence relation on ω^ω such that $x \mathrel{\mathsf{E}_0} y$ iff $x(k) = y(k)$ for all but finite k, and $[x]_{\mathsf{E}_0} = \{y \in \omega^\omega : x \mathrel{\mathsf{E}_0} y\}$ is the (countable) E_0-class of a real $x \in \omega^\omega$.

Let a *Groszek – Laver pair* be any OD pair of sets $X, Y \subseteq \omega^\omega$ such that neither of X, Y is separately OD. As demonstrated in [3], if $\langle x, y\rangle$ is a Sacks×Sacks generic pair of reals over L then their L-degrees $X = [x]_{\mathrm{L}} \cap \omega^\omega$ and $Y = [y]_{\mathrm{L}} \cap \omega^\omega$ form such a pair in $\mathrm{L}[x, y]$; the sets X, Y is this example are obviously uncountable.

THEOREM 2 ([2]). *There is a generic extension* $\mathrm{L}[a, b]$ *of* L *by reals* a, b *in which it is true that the* **countable** *sets* $[a]_{\mathsf{E}_0}$ *and* $[b]_{\mathsf{E}_0}$ *form a Groszek – Laver pair, and moreover the union* $[a]_{\mathsf{E}_0} \cup [b]_{\mathsf{E}_0}$ *is a* Π^1_2 *set.*

THEOREM 3 ([6]). *It is consistent with* **ZFC** *that there is a* Π^1_2 *set* $\emptyset \neq Q \subseteq \omega^\omega \times \omega^\omega$ *with* **countable** *cross-sections* $Q_x = \{y : \langle x, y\rangle \in Q\}$, $x \in \omega^\omega$, *non-uniformizable by any ROD set. In fact each cross-section* Q_x *in the example is a* E_0 *class.*

ROD = real-ordinal-definable. Typical examples of non-ROD-uniformizable sets, like $\{\langle x, y\rangle : y \notin \mathrm{L}[x]\}$ in the Solovay model, definitely have **un**countable cross-sections.

Let *analytically definable* mean the union $\bigcup_n \Sigma^1_n$ of all lightface definability classes Σ^1_n. The *full basis theorem* is the claim that any non-empty analytically definable set $X \subseteq \omega^\omega$ contains an analytically definable element. This is true assuming $\mathbf{V} = \mathbf{L}$, and generally assuming that there is an analytically definable wellordering of the reals. We prove that the implication is irreversible.

THEOREM 4 (with A. Enayat). *It is consistent with* **ZFC** *that the full basis theorem is true but there is no analytically definable wellordering of the reals.*

[1] ALI ENAYAT, *On the Leibniz-Mycielski axiom in set theory*, ***Fundamenta Mathematicae***, vol. 181 (2004), no. 3, pp. 215–231.

[2] M. Golshani, V. Kanovei, V. Lyubetsky, *A Groszek – Laver pair of undistinguishable* E_0 *classes*, ***Mathematical Logic Quarterly***, 2016, to appear.

[3] M. GROSZEK AND R. LAVER, *Finite groups of OD-conjugates*, ***Periodica Mathematica Hungarica***, vol. 18 (1987), pp. 87–97.

[4] RONALD JENSEN, *Definable sets of minimal degree*, ***Mathematical Logic and Foundations of Set Theory, Proceedings of an International Colloquium*** (Jerusalem 1968), (Yehoshua Bar-Hillel, editor), North-Holland, 1970, pp. 122–128.

[5] V. Kanovei, V. Lyubetsky, *A definable* E_0 *class containing no definable elements*, ***Archive for Mathematical Logic***, vol. 54 (2015), no. 5, pp. 711–723.

[6] V. Kanovei, V. Lyubetsky, *Counterexamples to countable-section* Π^1_2 *uniformization and* Π^1_3 *separation*, ***Annals of Pure and Applied Logic***, 2016, to appear.